AIAA JOURNAL
Vol. 32, No. 1, January 1994

Residual Flexibility Test Method for Verification of Constrained Structural Models

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A method is described for deriving constrained modes and frequencies from a reduced model based on a subset of the free-free modes plus the residual effects of neglected modes. The method involves a simple modification of the MacNeal and Rubin component mode representation to allow development of a verified constrained (fixed-base) structural model. Results for two spaceflight structures having translational boundary degrees of freedom show quick convergence of constrained modes using a measurable number of free-free modes plus the boundary partition of the residual flexibility matrix. This paper presents the free-free residual flexibility approach as an alternative test/analysis method when fixed-base testing proves impractical.

Nomenclature

A = transformation matrix for flexibility
F = external force vector
G = flexibility matrix
G_r = residual flexibility matrix
H = mass matrix associated with residual flexibility
K = stiffness matrix
$\bar{K}$ = reduced stiffness matrix
M = mass matrix
$\bar{M}$ = reduced mass matrix
m = number of retained modes plus boundary coordinates
N = number of coordinates in unreduced structure
n = number of retained or measured modes
n_b = number of boundary coordinates
q = generalized displacement vector
T = transformation matrix to reduce structure coordinates
u = physical structure displacement vector
Φ = mode shapes
ω,Ω = natural frequencies

Subscripts

b = boundary coordinates
c = constrained structure
f = flexible structure
i = interior coordinates
n = retained modes
R = rigid body
r = residual

Introduction

DEVELOPMENT of a dynamic model of a constrained structure having acceptable fidelity requires a modal survey test of the physical structure and subsequent improvement of the model using test data. Constrained-boundary or fixed-base testing has historically been the most common approach for verifying constrained mathematical models, since the boundary conditions of the test article are designed to match the actual constraints in service.

Unfortunately, there are a number of difficulties involved with fixed-base testing, making the approach impractical in some cases. As stated in Refs. 1 and 2, it is not possible to conduct a truly fixed-base test due to coupling between the test article and the fixture. In addition, it is often difficult to accurately simulate the actual boundary constraints, and the cost of designing and constructing the fixture may be prohibitive.

Alternate free-boundary test methods for deriving constrained modes have been investigated for use when measurement of fixed-base modes proves impractical or undesirable. The mass-additive technique[1,2] is an interesting approach that shows promise, though large sets of free-free modes are required for some structures. Blair and Vadlamudi[3] presented a useful method for model improvement where the minima of the interface response functions were used to assess model adequacy in the interface regions. This data in combination with the free-free mode set was used to obtain a Shuttle-constrained space telescope model for loads analysis.

In the current investigation, it was desirable to implement a technique that allows use of free-free mode sets small enough to be accurately measured and correlated with analysis, provides a means of determining the adequacy of the free-free model for deriving fixed-base modes, and makes the development of completely test-verified models possible. Based on previous studies, the residual flexibility approach[4-8] appears to have the greatest potential for meeting these requirements. The prior studies, with the exception of Ref. 7, focus on the residual flexibility method mainly in the context of component mode synthesis, though Rubin[5] derived the cantilever modes of a simple rod to demonstrate his technique. Reference 7 describes the correlation of a Space Shuttle payload model to measured free-free modes and residual flexibility of the interfaces. The residual flexibility model was then constrained as the flight hardware would be in the Shuttle orbiter.

The purpose of the present paper is to provide a detailed study of the free-free residual flexibility approach as a technique for deriving constrained structural modes when measured fixed-base modes are unavailable. The basic approach of Ref. 7 is utilized, but much additional work is done to assess the adequacy of various residual formulations for deriving constrained modes. Though the study is mainly analytical, some test data are presented. In the following sections, the method is described in detail and the governing equations are discussed. The method is then applied to the space station prototype module and a Space Shuttle payload (Figs. 1 and 2) to demonstrate verification of large constrained models for loads analyses. Results from application of the residual ap-

Presented as Paper 92-2325 at the AIAA/ASME/ASCE/AHS/ASC 33rd Structures, Structural Dynamics, and Materials Conference, Dallas, TX, April 13-15, 1992; received May 1, 1992; revision received March 12, 1993; accepted for publication April 8, 1993.

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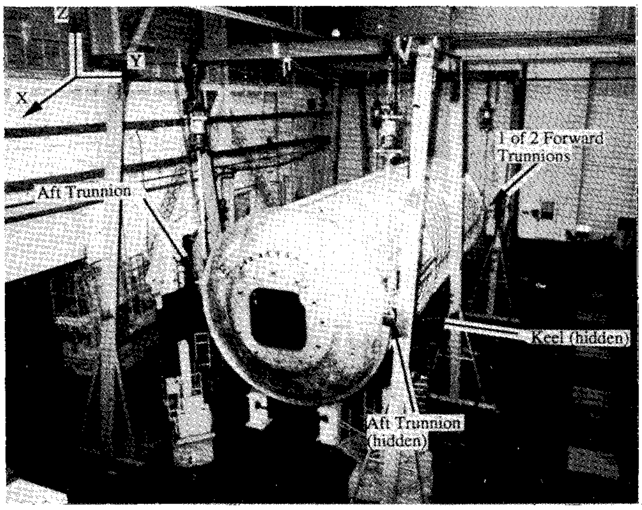

Fig. 1 Space station module prototype in free-boundary test configuration.

proach to the space station module (SSM) are compared to results of the mass-additive method described in Refs. 1 and 2. A comparison of analytical and test-derived residual functions for the SSM is also presented.

The approach taken in this paper is to constrain the residual flexibility model and compare the resulting fixed-base modes to those obtained from the full finite element model on which the residual model is based. In this manner, adequacy of the residual flexibility formulation for deriving constrained modes can be assessed. Significant differences in the two sets of constrained frequencies should indicate that either more terms of the residual flexibility matrix are needed or that an insufficient number of free-free modes have been used.

Background of Residual Flexibility Approach

The technique of using an approximation of the effects of neglected higher order modes, or residual modes, to improve the accuracy of reduced-basis mathematical models was first presented by MacNeal.[4] In MacNeal's paper, a substructure model derived from truncated modal properties was improved by including additional elements derived from first-order static approximations of the effects of higher modes. Rubin[5] used a special statics problem to derive an expression for residual flexibility in a form that is more easily applied in structural dynamic analyses. He showed that the flexible-body displacements for a structure can be written as a first-order approximation of residual effects,

$$u_f = GF = A^T G_c A F \tag{1}$$

where G is the free-free flexibility matrix. The constrained flexibility matrix is G_c and the flexibility transformation matrix is $A = I - M\Phi_R M_R^{-1}\Phi_R^T$. If the contribution of modes to be retained is removed from the deflection for the flexible structure, the residual flexibility matrix G_r results, as shown in Eq. (2):

$$u_{fr} = (G - G_n)F = G_r F \tag{2}$$

where $G_n = \Phi_n K_n^{-1}\Phi_n^T$ is the flexibility matrix corresponding to the retained modes. Rubin carried the procedure a step further to include second-order residual effects, but these will not be shown until later in the paper.

Both Rubin and MacNeal presented a stiffness matrix formulation for component mode synthesis using residual flexibility, providing for the first time a method in which complete experimental definition of a structure is possible. It appears that the only drawback to that representation for modal synthesis or structural loads analysis is the difference in format compared to the widely used Craig-Bampton method. Martinez, Carne, et al.[6] overcame this inconvenience by observing that the MacNeal and Rubin representation could be cast in a form similar to the Craig-Bampton formulation, thus facilitating practical use of the residual flexibility technique. As described in Ref. 6, structure displacements can be written in the form

$$u = \Phi q + G_{r_b} F_b = [\Phi \quad G_{r_b}] \begin{Bmatrix} q \\ F_b \end{Bmatrix} \tag{3}$$

where Φ is the $(N \times n)$ matrix of retained or measured modes and G_{r_b} is a partition of the $(N \times N)$ residual flexibility matrix defined in Eq. (2). If the displacements are partitioned into interior and boundary or interface degrees of freedom (DOF), Eq. (3) becomes

$$\begin{Bmatrix} u_i \\ u_b \end{Bmatrix} = \begin{bmatrix} \Phi_i & G_{r_{ib}} \\ \Phi_b & G_{r_{bb}} \end{bmatrix} \begin{Bmatrix} q \\ F_b \end{Bmatrix} \tag{4}$$

The lower partition of Eq. (4) is

$$u_b = \Phi_b q + G_{r_{bb}} F_b \tag{5}$$

which can be solved for the interface or boundary forces,

$$F_b = G_{r_{bb}}^{-1} u_b - G_{r_{bb}}^{-1} \Phi_b q \tag{6}$$

Substituting Eq. (6) into Eq. (4) yields

$$u_i = (\Phi_i - G_{r_{ib}} G_{r_{bb}}^{-1} \Phi_b) q + G_{r_{ib}} G_{r_{bb}}^{-1} u_b \tag{7}$$

This form of displacements for the structure results in modified free-boundary elastic modes [coefficient matrix of q in Eq. (7)] and modified residual attachment modes (coefficient of u_b). Assembly of Eq. (7) with the identity $u_b = u_b$ gives the final set of equations for the structure displacements,

$$\begin{Bmatrix} u_i \\ u_b \end{Bmatrix} = \begin{bmatrix} \Phi_i - G_{r_{ib}} G_{r_{bb}}^{-1} \Phi_b & G_{r_{ib}} G_{r_{bb}}^{-1} \\ 0 & I \end{bmatrix} \begin{Bmatrix} q \\ u_b \end{Bmatrix} = T \begin{Bmatrix} q \\ u_b \end{Bmatrix} \tag{8}$$

where T is an $(N \times m)$ matrix and $m = n + n_b$, the number of retained modes plus the number of boundary DOF.

The form of the transformation matrix in Eq. (8) is similar to that employed by Craig and Bampton[9] and is therefore more appealing than earlier formulations of the residual flexibility method to engineers accustomed to using the Craig-

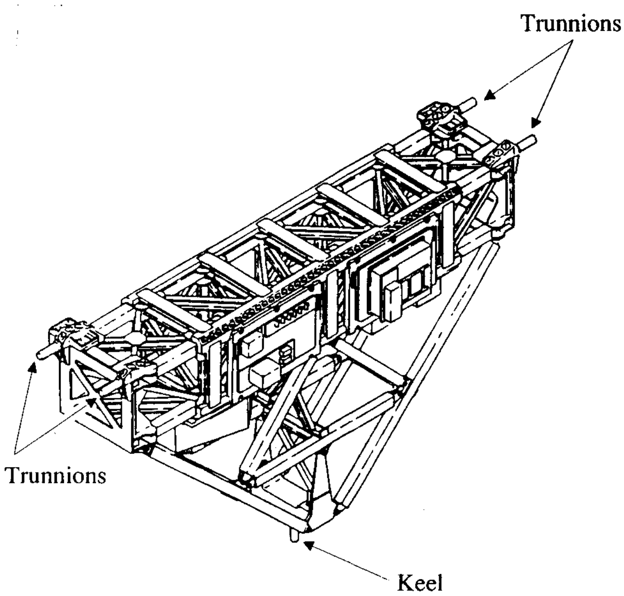

Fig. 2 Shuttle payload with orbiter connections.

Bampton approach. Kammer and Baker[10] showed that the residual flexibility approach in the form expressed in Ref. 6, Eq. (8), is statically equivalent to the Craig-Bampton formulation. It is also shown in Ref. 10 that under certain conditions the two representations are equivalent for the general dynamics problem, and Craig[11] presents a thorough discussion of the equivalence of component mode representations. In the present paper, the representation in Eq. (8) is demonstrated as a general test method for verifying structural dynamic models having constrained boundaries. For this reason, the current study differs from much of the earlier work with the residual flexibility method which emphasized its use for component synthesis. In the following sections, the implementation is described analytically and demonstrated for the space station prototype module and a Shuttle payload.

Implementation of the Technique for Test Verification of Constrained Models

Starting with the undamped equation of motion for a structure,

$$M\ddot{u} + Ku = F \tag{9}$$

the partitioned form of the equation is written

$$\begin{bmatrix} M_{ii} & M_{ib} \\ M_{bi} & M_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{u}_i \\ \ddot{u}_b \end{Bmatrix} + \begin{bmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{bmatrix} \begin{Bmatrix} u_i \\ u_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_b \end{Bmatrix} \tag{10}$$

The corresponding partitioned form of the residual flexibility matrix is

$$G_r = \begin{bmatrix} G_{r_{ii}} & G_{r_{ib}} \\ G_{r_{bi}} & G_{r_{bb}} \end{bmatrix} \tag{11}$$

The desired elements of G_r are to be obtained using frequency response measurements of the free-free test article for the connect coordinates and shaker drive points[5,7,8] or computed using Eqs. (1) and (2). If the residual values are calculated, they must be checked using measured data before the model is considered verified. The retained natural frequencies and mode shapes, ω_n^2 and Φ_n, are to be obtained from a free-boundary modal test or from a test-correlated model, and correspond to subsets of the eigenvalues and eigenvectors of Eq. (10) with $F = 0$. Applying the transformation defined in Eq. (8) to Eq. (10), the reduced equation of motion becomes

$$\bar{M}\begin{Bmatrix} \ddot{q} \\ \ddot{u}_b \end{Bmatrix} + \bar{K}\begin{Bmatrix} q \\ u_b \end{Bmatrix} = T^T\begin{Bmatrix} 0 \\ F_b \end{Bmatrix} \tag{12}$$

where $\bar{M} = T^TMT$ and $\bar{K} = T^TKT$. It is shown in Ref. 6 that

$$\bar{M} = \begin{bmatrix} I_{nn} + \Phi_{nb}^T J_{bb} \Phi_{nb} & -\Phi_{nb}^T J_{bb} \\ \text{sym} & J_{bb} \end{bmatrix}$$
$$\bar{K} = \begin{bmatrix} \Omega_{nn}^2 + \Phi_{nb}^T G_{r_{bb}}^{-1} \Phi_{nb} & -\Phi_{nb}^T G_{r_{bb}}^{-1} \\ \text{sym} & G_{r_{bb}}^{-1} \end{bmatrix} \tag{13}$$

where Ω_{nn} is the diagonal matrix of retained or measured frequencies ω_n, and Φ_{nb} is the boundary partition of the retained modes. Also in Eq. (13), $J_{bb} = G_{r_{bb}}^{-1} H_{bb} G_{r_{bb}}^{-1}$ and $H_{bb} = G_{r_b}^T M G_{r_b}$, where $G_{r_b} = [G_{r_{ib}}\ G_{r_{bb}}]^T$ as shown in Eq. (4). The residual mass effects corresponding to residual flexibility are contained in H_{bb}. The mass and stiffness matrices in Eq. (13) represent the full-residual or Rubin method ("full residual" in this paper means that only residual damping effects are neglected). If the residual mass effects are neglected, the MacNeal formulation is obtained and the generalized mass becomes a unity matrix.

The constrained model is obtained by setting boundary displacements in Eq. (12) equal to zero, yielding

$$\bar{M}_{nn}\ddot{q} + \bar{K}_{nn}q = 0 \tag{14}$$

where $\bar{M}_{nn} = [I_{nn} + \Phi_{nb}^T J_{bb} \Phi_{nb}]$ and $\bar{K}_{nn} = [\Omega_{nn}^2 + \Phi_{nb}^T G_{r_{bb}}^{-1} \Phi_{nb}]$, and both matrices are $(n \times n)$. The eigenvalues ω_c^2 calculated from Eq. (14) are the constrained frequencies, and the constrained modes are obtained by assembling the eigenvectors from Eq. (14), Φ_{nn}, into an $(m \times n)$ matrix and premultiplying by T from Eq. (8):

$$\Phi_c = T\begin{bmatrix} \Phi_{nn} \\ 0 \end{bmatrix} \tag{15}$$

Since T is $(N \times m)$ and the partitioned mode shape matrix is $(m \times n)$, an $(N \times n)$ matrix of constrained modes is obtained. The frequencies and mode shapes for the constrained structure, ω_c and Φ_c, provide a check on the adequacy of the residual flexibility representation when they are compared to "exact" values obtained from the full finite element model, Eq. (10). Results of the solution of the equations discussed in this section to derive constrained modes and frequencies for large complex structures are presented in the following section.

Results for Spaceflight Structures

Space Station Module Prototype

A study was performed to determine the convergence characteristics of the residual flexibility approach for realistic spaceflight hardware having translational boundary DOF. The space station module (SSM) prototype is shown in Fig. 1 in a free-boundary test configuration. Most of the results in the study were obtained using 402-DOF finite element models that had been partially updated using only free-free test modes and frequencies. Only the results in Fig. 6 refer to model updates based on residual measurements. Modal testing of the SSM is described in Ref. 12, and the test/model correlation is discussed in Refs. 13–15. Table 1, from Ref. 13, shows a comparison of free-free modes obtained from test and from a model updated using free-free test modes.

Convergence of constrained modes for the SSM using free-free normal modes and full-residual effects (mass and flexibility) is shown in Fig. 3. The error between exact and derived constrained frequencies is shown as a function of the number of free-free modes used in the solution. The derived modes come from Eqs. (14) and (15), and the exact constrained modes are obtained from a full finite element model correlated to free-free test modes [Eq. (10)]. For comparison of methods examined in this study, "convergence" of the constrained modes is defined to have been achieved when the error between exact and derived constrained frequencies is 1% or less. It can be seen that for this complex shell structure, which is considered a worst-case application due to the existence of many localized modes, convergence is achieved with approximately 20 free-free modes. In Fig. 3 the four constrained

Table 1 Comparison of space station module free-free modes from test and partially updated model

Test mode no.	Test frequency	Model frequency	Cross correlation	Mode description
1	15.73	18.56	0.93	Keel fitting, x
2	19.38	19.09	0.89	Fwd. trunnion, x
3	19.74	19.05	0.873	Fwd. trunnion, x
4	20.53	20.83	0.825	First ovalling
5	20.64	21.41	0.936	First ovalling
6	27.61	27.05	0.915	Second ovalling
7	27.85	27.73	0.924	Second ovalling
8	31.52	33.90	0.942	Shell
9	34.75	39.03	0.771	Shell, spreader bar
10	36.12	40.53	0.471	Shell, spreader bar
11	37.03	42.68	0.591	Fwd. shell
12	40.11	42.05	0.374	General shell
13	40.58	45.80	0.380	Gen. shell, aft trun., x
14	41.19	42.05	0.374	Gen. shell, aft trun., x
15	42.53	41.81	0.647	General shell

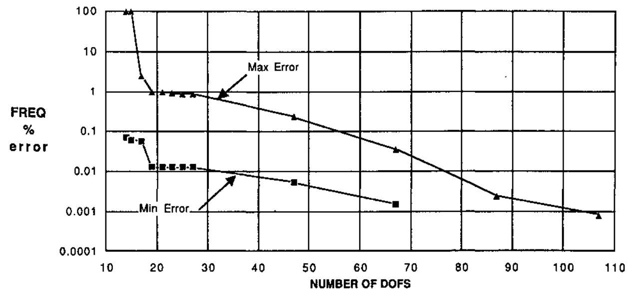

Fig. 3 Frequency error between exact and derived constrained modes of space station module using full-residual effects (Rubin method).

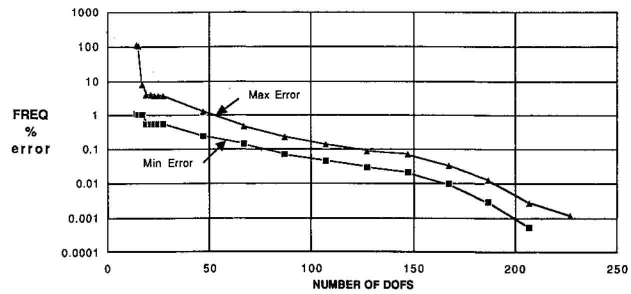

Fig. 4 Frequency error between exact and derived constrained modes of space station module using only residual flexibility (MacNeal method).

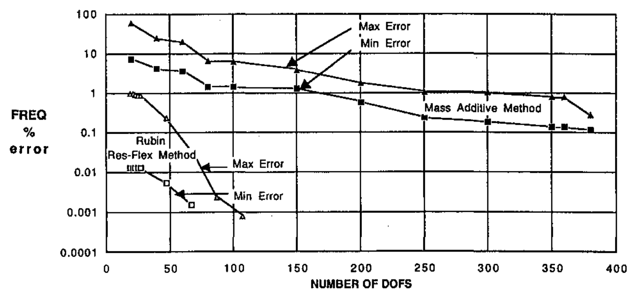

Fig. 5 Comparison of constrained-frequency convergence rates for full-residual method and mass-additive method applied to space station module.

modes causing the highest structural loads are the basis of comparison, and the maximum error is the largest difference observed between exact and derived frequencies of those four modes, while the minimum error is the smallest difference observed.

In Fig. 4 the convergence of constrained modes is shown for the MacNeal method (including residual flexibility but neglecting residual mass effects). It can be seen that this approach converges to within 1% error on frequency with approximately 50 free-free modes. For this shell structure the full-residual (Rubin) method would be required due to the difficulty in correlating the model to test data for this many modes. Figure 5 shows a comparison of the full-residual method and the mass-additive modal test method described in Refs. 1 and 2. In the mass-additive approach, the boundaries are mass loaded to exercise the structure load paths and thereby obtain free-free modes that can be superimposed to derive constrained modes. It can be seen in Fig. 5 that the full-residual method converges to within 1% error on frequency with about 20 modes (also shown in Fig. 3) whereas the mass-additive approach requires approximately 250 modes for the same degree of convergence. It was stated in Ref. 1 that the mass-additive method is insufficient for deriving constrained modes of the space station module due to the requirement of a large free-free mode set. Results in this paper show that the full-residual approach works very well for the SSM.

It must be emphasized at this point that the full partition G_{r_b} of the residual flexibility matrix was used to obtain the results in Figs. 3–5, and that none of these residuals were test verified. It has been suggested[5] that part or all of the boundary partition $G_{r_{bb}}$ can be measured for model verification while neglecting the other partitions. This approach has appeal because it is impractical to measure the full boundary partition G_{r_b} and because the residual flexibilities are typically largest near the boundary constraint or test drive point DOF.[7,13] In Table 2 the partition $G_{r_{bb}}$ is shown, computed using a model of the SSM correlated to test modes. Analysis has been performed to assess the accuracy of derived constrained modes for two cases where all of the residuals are neglected except the diagonals of $G_{r_{bb}}$ and the full partition $G_{r_{bb}}$. Results for these two cases are shown in Table 3 for the MacNeal method with 50 free-free modes. For a solution using only diagonals of the boundary partition, the error between exact and derived fundamental frequencies is greater than 5%, although the error is smaller for the other frequencies. Therefore, a MacNeal solution for the space station module using only diagonals of $G_{r_{bb}}$ is insufficient to meet the 1% frequency error criterion used consistently for the comparison of methods in this study. However, it can also be seen from Table 3 that a MacNeal solution using the full boundary partition of the residual flexibility matrix does meet the convergence requirement.

As described by Blair,[13–15] experimental boundary residual functions for the SSM were obtained for use in model correlation. Values of these acceleration/force functions at frequencies well above the minima correspond to diagonals of $G_{r_{bb}}$, since high-frequency residual mass effects were not included. Residual functions for two boundary drive points or constraint DOF are shown in Fig. 6 (from Ref. 13) with a comparison drawn to results for the correlated model. It is noted that

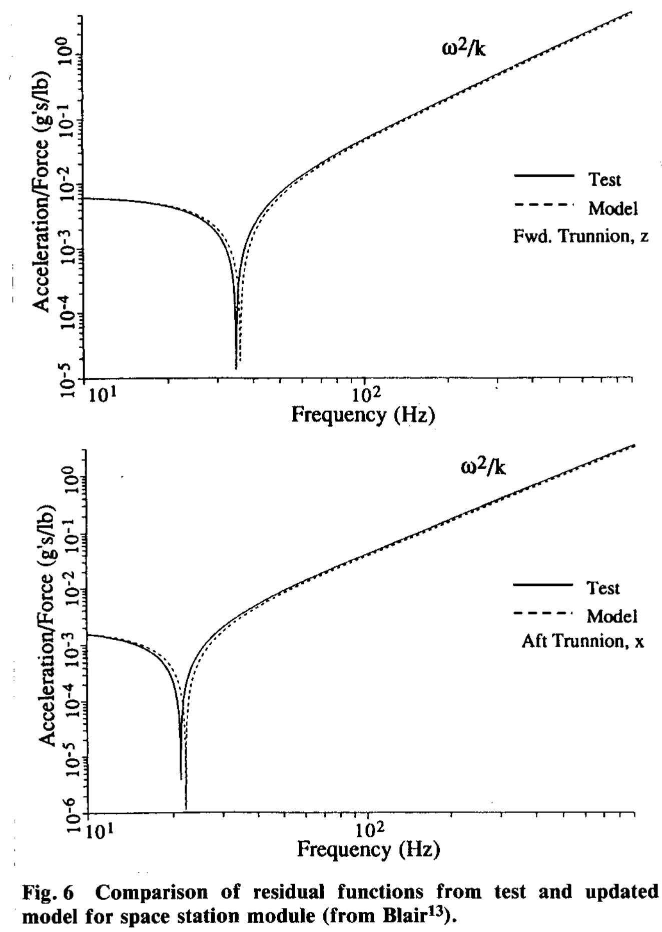

Fig. 6 Comparison of residual functions from test and updated model for space station module (from Blair[13]).

Table 2 Boundary partition of analytical residual flexibility matrix for space station module with 50 free-free modes in MacNeal method (units: 1.0E − 6 in./lb)

5.4825	−0.49316	−0.24397	−0.03931	0.76427	−0.03028	−0.32043
−0.49316	**5.4856**	0.76432	−0.03043	−0.24396	−0.03931	0.32066
−0.24397	0.76432	**3.8773**	−0.13622	−0.34212	−0.04565	0.005328
−0.03931	−0.03043	−0.13622	**4.5370**	−0.04547	−0.32896	0.33152
0.76427	−0.24396	−0.34212	−0.04547	**3.8805**	−0.13528	−0.00423
−0.03028	−0.03931	−0.04565	−0.32896	−0.13528	**4.5369**	−0.33139
−0.32043	0.32066	0.005328	0.33152	−0.00423	−0.33139	**7.6590**

Table 3 Constrained frequencies for space station module retaining only boundary partition of residual flexibility matrix (MacNeal method, 50 modes)

Mode no.	Exact freq.	Derived frequencies and percent error				Mode correlation with exact	
		Diagonals of $G_{r_{bb}}$	Percent error	Full $G_{r_{bb}}$	Percent error	Diagonals of $G_{r_{bb}}$	Full $G_{r_{bb}}$
1	9.7913	10.3058	−5.255	9.8201	−0.294	0.99616	0.99646
2	13.3956	13.2520	1.072	13.4245	−0.216	0.99658	0.99680
3	18.3037	18.1999	0.567	18.3486	−0.245	0.99604	0.99640
4	22.3811	22.3818	−0.003	22.3819	−0.004	0.99990	0.99993
5	22.5663	22.5660	0.001	22.5664	−0.0004	0.99998	0.99999
6	23.7289	23.7140	0.063	23.7726	−0.184	0.99728	0.99748
7	25.7756	25.6887	0.337	25.9876	−0.822	0.98690	0.98721
8	28.4371	28.5814	−0.507	28.5133	−0.268	0.99594	0.99623
9	29.5605	29.5398	0.070	29.6378	−0.261	0.99596	0.99526
10	31.2879	31.3232	−0.113	31.3181	−0.097	0.99707	0.99751

this model was correlated to both free-free modes and boundary residual functions from test results, whereas the other models used in this study were updated using test modes alone.

Experimental residual functions in Fig. 6 were obtained as described by Ewins[16] and as commonly obtained in commercially available modal testing/analysis software. Using all of the modes in the frequency range of interest, modal frequency response functions (FRF) were synthesized or theoretically regenerated, and then subtracted from the corresponding fully measured FRF. This procedure, applied to each boundary FRF at the lowest frequencies in the test bandwidth estimates a residual inertance constant, or mass line with a zero slope on the log-log scale. At the highest frequencies in the bandwidth, the procedure estimates a residual flexibility constant, or a stiffness line with slope of 2 on the log-log scale. These two residual terms, when added to the synthesized FRF, bring it into close agreement with the full measured data, and the combination of the mass and stiffness lines yields a residual function of the form shown in Fig. 6. Related procedures were used with the correlated model to obtain analytical residual functions.[13-15] The value of each function above the minimum or zero is ω^2/k for a given frequency. A value of $1/k$, or residual flexibility, for each boundary DOF can be calculated by choosing a point on the straight-line portion of the residual function and dividing the value of the function by ω^2.

It is noted that the residual functions in Fig. 6 are different in two respects from those that would be obtained using Rubin's approach. First, the Rubin method retains the rigid body modes as part of the free-free mode set and therefore does not have the low-frequency residual inertance evident in the functions discussed in this paper. Second, Rubin's approach yields residual functions with high-frequency residual mass effects that are not present in Fig. 6. The consequence of this second difference is that determination of a residual flexibility value using Rubin acceleration/force residual functions requires curve fitting with a fourth-order polynomial containing both residual flexibility and residual mass.

Space Shuttle Payload

Residual flexibility analysis has also been performed for the material science laboratory (MSL) payload, and the results

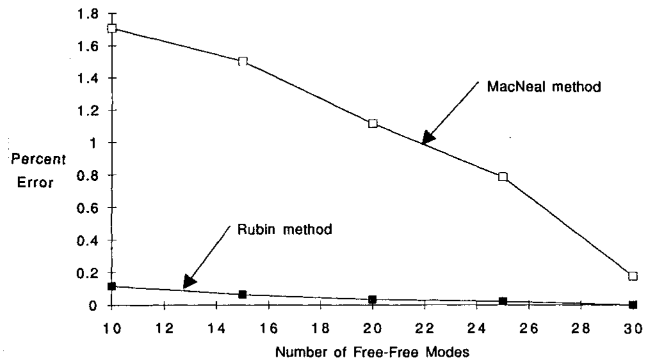

Fig. 7 Comparison of constrained-frequency convergence rates for Rubin and MacNeal methods applied to Shuttle payload (full partition of residual flexibility matrix).

will be discussed briefly. Analysis of this structure is of interest due to the vast differences in geometry, mass, and stiffness compared to the space station module. As shown in Fig. 2, this payload is a truss-like structure with stiff supports for the interfaces (trunnions and keel). The MSL flown had numerous attachments and experiments, but these are not shown in Fig. 2. As was the case for most of the space station module analysis, calculations for MSL were performed using a model correlated to free-free test modes only. However, the purpose of these calculations is to provide information on the relative constrained frequency errors obtained in the use of various residual formulations, and the models are considered sufficiently accurate.

A comparison of the error between exact and derived constrained frequencies for the MacNeal method and full-residual (Rubin) method is shown in Fig. 7. Again, the exact results are obtained using a full finite element model. In both cases the entire partition G_{r_b} of the residual flexibility matrix was used. Of course, the full-residual method converges more quickly, but the MacNeal method works very well for this structure. It is noted that only 22 modes are required for a 1% frequency error. In Table 4 a comparison of exact and derived frequencies is shown for all residuals neglected except diagonals of

Table 4 Constrained frequencies for Shuttle payload retaining only boundary partition of residual flexibility matrix (MacNeal method, 25 modes)

Mode no.	Exact freq.	Derived frequencies and percent error			
		Diagonals of $G_{r_{bb}}$	Percent error	Full $G_{r_{bb}}$	Percent error
1	13.6779	13.4316	1.801	13.6937	−0.116
2	18.7771	19.4164	−3.405	18.8190	−0.223
3	21.3016	21.8060	−2.368	21.3480	−0.218
4	29.0195	30.5621	−5.316	29.2473	−0.785
5	30.5975	30.7681	−0.558	30.6326	−0.115
6	31.4175	32.3364	−2.925	31.4789	−0.195
7	37.0519	37.0962	−0.120	37.0664	−0.039
8	37.3099	37.4297	−0.321	37.3546	−0.120
9	43.2756	42.4517	1.904	43.3983	−0.284
10	47.2276	46.8069	0.891	47.3344	−0.226

$G_{r_{bb}}$ and full partition $G_{r_{bb}}$. The MacNeal approach was used, and 25 free-free modes were retained for each case. As for the space station module, it is seen that the full partition $G_{r_{bb}}$ is needed to achieve a 1% frequency error if residual mass is not included.

Summary and Discussion

The application of the residual flexibility method to test verification of constrained models has been demonstrated. Derivation of constrained modes and frequencies using free-boundary modes and residual flexibility has been performed for large complex structures. Convergence of constrained frequencies to within a 1% error was achieved using 20 modes in the Rubin method for the space station module, and using 22 modes in the MacNeal method for a Shuttle payload with stiff interface structures. These results show that residual flexibility must be accompanied by residual mass effects for some structures, and that a complete pretest analysis must be done to determine which residual effects are required and to determine the number of free-free modes needed in a test. However, this paper has shown analytically, and to a limited degree experimentally (Fig. 6), that the residual flexibility approach has great potential as a method for deriving constrained modes when fixed-base testing is not feasible.

Some investigators have expressed concern over the difficulty in performing the required frequency response measurements for the method. This concern is justified for a number of reasons. Two of these difficulties are well-described by Blair in Ref. 15. First, residual flexibilities are very small numbers, typically on the order of $1.0E-6$ in./lb for diagonal terms, and orders of magnitude smaller for off-diagonal values. This poses a difficulty in obtaining accurate and noise-free measurements, especially for points removed from the excitation source. A second difficulty encountered in residual measurements lies in obtaining a clean residual function in the process of subtracting synthesized modal data from a measured response function. Inaccuracies occur because modes are not subtracted exactly, but only to the accuracy of the curve fits for each mode; these errors are compounded with increasing distance from the excitation point. A third difficulty is the measurement of rotational residual terms. It is well-known that rotational measurements are usually not attempted in modal testing. Several methods have been used with limited success, but this technology is still in development.[16] Mainly for this reason, the residual flexibility method has not been widely used for general boundary configurations. Fourth, it has been shown that misalignment of shakers can cause significant errors in the measured frequency response functions; and finally, a large number of measurements could be required to determine the desired elements of the residual flexibility matrix.

Although this paper has demonstrated the use of the residual flexibility approach for deriving constrained modes of large structures with translational boundary DOF, and has presented residual test data, much work remains to be done in the area of measurements. Curve-fitting methods in modal testing need to be developed that are better suited to residual measurements so that cleaner data can be obtained. Significant effort needs to be put in to improving rotational measurements to make the approach more applicable to structures having general boundary conditions. However, it is felt by many involved in modal analysis and testing that the advantages of the residual flexibility method merit continued development of the technique. Though problems remain to be solved, the residual flexibility approach allows the cost and difficulty associated with design, construction, and use of large test fixtures to be avoided.

Acknowledgments

Test data for this study was provided by the Dynamics Test Branch at Marshall Space Flight Center under the direction of A. D. Coleman. Residual functions shown in Fig. 6 were obtained by Mark Blair of Lockheed Missiles and Space Company under NASA Contract NAS8-38251.

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